An Impossibility Result on Methodological Individualism

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In the philosophy of the social sciences, individualism is the methodological precept that any social phenomenon is to be explained ultimately in terms of the actions and interactions of individuals. Ideally, such an explanation of a social phenomenon includes a set of background laws and a set of bridge laws that translate the social statements into individualistic statements. We show that there are no such bridge laws for collective deontic admissibility: it is impossible to define collective deontic admissibility in terms of a multi-modal logic that is widely used to model actions, omissions, abilities, obligations, prohibitions, and permissions of finitely many individual agents.

These philosophical considerations can be made precise by adopting a logical point of view. A necessary condition for a bridge law is that the social statement to be translated must be logically or at least nomologically equivalent to its individualistic translation: they must be true under exactly the same circumstances, where the range of relevant circumstances might be restricted by individualistic background laws.[1] Accordingly, if we wish to prove that there is no bridge law that translates a particular social statement S into a specific individualistic language $\mathfrak{L}_{ind}$, it suffices to show the following: there are no individualistic statements A and B in $\mathfrak{L}_{ind}$ such that S and A are equivalent modulo B, where B can be thought of as a finite conjunction of background laws.

The social statements that we focus on in this paper are collective deontic admissibility statements of the form "Group $\mathcal{G}$ of agents performs a deontically ad-

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[1] Two statements are conceptually co-extensive if and only if their extensions are the same in all possible worlds. Two statements are nomologically co-extensive if and only if their extensions are the same in all possible worlds where the relevant background laws hold.

missible group action". The notion of collective admissibility is central to formal studies of forward-looking and backward-looking collective moral responsibility and of collective rationality. It is also an indispensable notion in team-reasoning and we-reasoning accounts of cooperation.

The proof of our impossibility result is organized as follows. First, we define the formal social language $\mathfrak{L}$ (of which the individualistic language $\mathfrak{L}_{ind}$ is a sublanguage) and interpret the sentences of that language by way of what we call *deontic game models*. Second, we introduce the notion of individualistic bisimulation and define two constructions that transform a given deontic game model into deontic game models that validate exactly the same individualistic formulas as the given model but give different truth values to collective deontic admissibility statements. Finally, this formal apparatus enables us to prove our impossibility result: there are no bridge laws translating collective admissibility statements into the individualistic language $\mathfrak{L}_{ind}$.

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A Some Formal Details

A.1 Language

Let $\mathfrak{P}$ be a countable set of atomic propositions and let $\mathcal{N}$ be a finite set of individual agents. We use p and q as variables for atomic propositions, i as a variable for individual agents, and $\mathcal{F}$ and $\mathcal{G}$ as variables for sets of individual agents. The social language $\mathfrak{L}$ is given by:

$$\phi := p \mid \star_{\mathcal{G}} \mid \neg\phi \mid \phi \wedge \phi \mid \Box\phi \mid [i]\phi.$$

The meaning of these sentences will be rigorously cached out in terms of deontic game models. At this point, it may be helpful to indicate that $\star_{\mathcal{G}}$ is the statement that "group $\mathcal{G}$ performs an admissible group action". The individualistic sublanguage $\mathfrak{L}_{ind}$ is given by:

$$\phi := p \mid \star_i \mid \neg\phi \mid \phi \wedge \phi \mid \Box\phi \mid [i]\phi.$$

A.2 Deontic Game Models

Definition 1 (Deontic Game Model)**.** *A* deontic game model M *is a quadruple* $\langle \mathcal{N}, (A_i), d, v \rangle$*, where* $\mathcal{N}$ *is a finite set of individual agents, for each agent* i *in* $\mathcal{N}$ *it holds that* A_i *is a non-empty and finite set of actions available to agent* i*,* $d : A \to \{0, 1\}$ *is a deontic ideality function such that there is at least one* a *in* A *with* $d(a) = 1$*, and* $v : \mathfrak{P} \to \wp(A)$ *is a valuation function.*

Definition 2 (Dominance)**.** *Let* $M = \langle \mathcal{N}, (A_i), d, v \rangle$ *be a deontic game model. Let* $\mathcal{G} \subseteq \mathcal{N}$ *be a set of individual agents. Let* $a_{\mathcal{G}}, b_{\mathcal{G}} \in A_{\mathcal{G}}$*. Then*

$$a_{\mathcal{G}} \succeq b_{\mathcal{G}} \quad \textit{iff} \quad \textit{for all } c_{-\mathcal{G}} \in A_{-\mathcal{G}} \textit{ it holds that } d(a_{\mathcal{G}}, c_{-\mathcal{G}}) \geq d(b_{\mathcal{G}}, c_{-\mathcal{G}}).$$

Definition 3 (Deontic Admissibility)**.** *Let* $M = \langle \mathcal{N}, (A_i), d, v \rangle$ *be a deontic game model. Let* $\mathcal{G} \subseteq \mathcal{N}$ *be a set of individual agents. Then the set of* $\mathcal{G}$*'s* deontically admissible actions *in* M*, denoted by* $Adm_M(\mathcal{G})$*, is given by*

$$Adm_M(\mathcal{G}) \quad = \quad \{a_{\mathcal{G}} \in A_{\mathcal{G}} : \textit{there is no } b_{\mathcal{G}} \in A_{\mathcal{G}} \textit{ such that } b_{\mathcal{G}} \succ a_{\mathcal{G}}\}.$$

Definition 4 (Truth-Conditions)**.** *Let* $M = \langle \mathcal{N}, (A_i), d, v \rangle$ *be a deontic game model. Let* $i \in \mathcal{N}$ *be an individual agent and let* $\mathcal{G} \subseteq \mathcal{N}$ *be a set of individual agents. Let* $a, b \in A$ *be action profiles. Let* $p \in \mathfrak{P}$ *be an atomic formula and* $\phi, \psi \in \mathfrak{L}$ *be arbitrary formulas. Then*

$$\begin{array}{lll}
(M, a) \models p & \textit{iff} & a \in v(p) \\
(M, a) \models \star_{\mathcal{G}} & \textit{iff} & a_{\mathcal{G}} \in Adm_M(\mathcal{G}) \\
(M, a) \models \neg\phi & \textit{iff} & (M, a) \not\models \phi \\
(M, a) \models \phi \wedge \psi & \textit{iff} & (M, a) \models \phi \textit{ and } (M, a) \models \psi \\
(M, a) \models \Box\phi & \textit{iff} & \textit{for all } b \in A \textit{ it holds that } (M, b) \models \phi \\
(M, a) \models [i]\phi & \textit{iff} & \textit{for all } b \in A \textit{ with } b_i = a_i \textit{ it holds that } (M, b) \models \phi.
\end{array}$$

Given a deontic game model M, we write $M \models \phi$, if for all action profiles a in A it holds that $(M, a) \models \phi$. A formula ϕ is *valid* (notation: $\models \phi$), if for all deontic game models M it holds that $M \models \phi$. A formula ϕ is *satisfiable* if $\not\models \neg\phi$, that is, if there exists a deontic game model M and action profile a such that $(M, a) \models \phi$.

A.3 The Impossibility Results

There are no individualistic statements A and B in $\mathfrak{L}_{ind}$ such that A and the statement that the group performs a deontically admissible group action are equivalent modulo the background law B, except if B is a contradiction:

Theorem 1. *Let $\mathcal{G} \subseteq \mathcal{N}$ be a non-empty and non-singleton group of agents. Then there are no $\phi_A, \phi_B \in \mathfrak{L}_{ind}$ such that $\not\models \neg\phi_B$ and $\models \phi_B \rightarrow (\phi_A \leftrightarrow \star_{\mathcal{G}})$.*

Corollary 1. *Let $\mathcal{G} \subseteq \mathcal{N}$ be a non-empty and non-singleton group of agents. Then there is no $\phi_A \in \mathfrak{L}_{ind}$ such that $\models \star_{\mathcal{G}} \leftrightarrow \phi_A$.*