Quantifying over the indexes of obligation operators

Generalized sentences involving deontic operators are of paramount importance in ethical reasoning. Consider 'All parents have an obligation to take care of their children', 'It is obligatory for all those who borrowed books that the books are returned to the library' and 'Everyone ought to maximise expected utility'. This last sentence is a proper *general obligation* [2], meaning that the obligation expressed in it holds for everyone. On the other hand, the obligations in the first two sentences are restricted to persons satisfying certain conditions (i.c. 'being a parent' and 'having borrowed books'). We call sentences exhibiting this property *restricted obligations*.

There are two additional properties that are illustrated by these three sentences. Firstly, all three of them concern universally quantified *personal obligations*. This means that they express an obligation that all persons, or all persons in a restricted group, have. This can be contrasted with *situational obligations* that describe the way a situation ought to be. Personal obligations can also be contrasted with *directed obligations* that describe not only for whom the obligation holds, but also to whom the obligation holds. These are the bearer and the counterparty of the obligation [3].

Secondly, some personal obligations are *agent-implicating* [5]: the subject of the obligation is also the agent of the obligatory action. The first and third sentences contain agent-implicating obligations, but not the second. According to the second sentence, the obligation on the borrowers of books can be fulfilled no matter who returns the books. This mirrors McNamara's example of personal non-agential obligations [6].

In spite of their central role in ethical reasoning, general and restricted obligations have mostly been ignored in deontic logic. Notable exceptions are the work by Hansson [2] and by Herrestad and Krogh [3]. Both of those approaches are, however, propositional. As a result they fail to capture both restricted obligations and the fact that many, but not all, personal obligations are agent-implicating. Capturing these two features seems to call for first-order logic.

However, simply extending standard deontic logic (SDL) with the tools of first-order logic is not sufficient to capture reasoning with general and restricted obligations. Consider the sentences 'All parents have an obligation to take care of their children' and 'Albert is a parent of Ben, but does not take care of Ben'. From this we derive 'Albert has violated his obligation to take care of Ben'. In a simple extension of SDL with quantification we could formalise the premises as:

(1) $(\forall x)(\forall y)(Pyx \to \mathsf{O}Cyx)$
(2) $Pab \wedge \neg Cab$

By universal instantiation (UI) and the propositional calculus (PC) we can then derive

(3) $\mathsf{O}Cab \wedge \neg Cab$

denoting the violation of a rule. However, this does not express that it was Albert who violated his obligation. To see this, let Tba stand for 'Ben is taken care of by Albert' and suppose that Cab and Tba are logically equivalent, i.e. in every possible world they are either both true or both false. But then, whenever (3) is true, we also have:

(4) $\mathsf{O}Tba \wedge \neg Tba$

If (3) were to indicate that Albert violated his obligation to take care of Ben, then (4) would indicate that Ben has violated his obligation to be taken care of by Albert.

To indicate that it is Albert, and not Ben, whose obligation has been violated, we suggest indexing the O-operator in (3) with the constant a. A consequence of this suggestion is that the formalisation of 'All parents have an obligation to take care of their children' must include an O-operator indexed by a variable:

(5) $(\forall x)(\forall y)(Pyx \to \mathsf{O}_y Cyx)$

We are thus in need of a logic that can handle quantification over modal operators indexed by terms of the language.

The goal of our talk is to present such a logic. This logic, term-modal deontic logic (TMDL), is a conservative extension of SDL. The language of TMDL includes identity and O-operators with two, one and no indexes to represent directed, personal and situational obligations. The models of TMDL use constant domains and treat constants as rigid designators. We will present a sound and strongly complete Hilbert-style axiomatisation of TMDL. We will show that this allows us to capture reasoning with general and restricted obligations, both when they are and when they are not agent-implicating.

Our development of TMDL builds on the term-modal logics presented by Fitting et al. [1]. Fitting et al. present sound and complete sequent calculi and tableau systems for these term-modal logics. However, their specific application context calls for increasing domain semantics, whereas we will argue that in a deontic context constant domain semantics are more appropriate. Another difference between their term-modal logics and TMDL is that their language does not include identity, nor modal operators with two or no indexes.

In TMDL these operators are used to represent situational and directed obligations. In our talk we will discuss two logical properties of these two kinds of obligation. First their interaction with personal obligations and secondly the possible restriction that all the directed obligations of one person must be jointly satisfiable [3].

We will end our talk by discussing how TMDL can be used to capture Hohfeldian rights [4]. Earlier work on Hohfeldian rights in deontic logic was focussed on *paucital* rights. Suppose for example that Ann and Suzie have signed a labour contract. Then Ann will have the paucital right to receive a wage from Suzie, whereas Suzie will have the corresponding obligation to Ann to pay her a wage. We will show that TMDL can capture not only *paucital*, but also *multital* rights. An example of such a multital right is the right that Bob has to sell a painting he owns. According to Hohfeld this corresponds to an obligation on all others not to sell the painting. We propose that this right and the corresponding obligation can be formalized as $\mathsf{P}_b Sb \wedge (\forall x)(x \neq b \rightarrow \mathsf{O}_x \neg Sx)$.

References

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